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1993 J. Phys. A: Math. Gen. 26 2883

(http://iopscience.iop.org/0305-4470/26/12/025)

J. Phys. A: Math. Gen. 26 (1993) 2883–2893. Printed in the UK

Spherical random-field systems with long-range interactions: general results and application to the Coulomb glass

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Received 14 January 1993

Abstract. A classical spherical random-field Hamiltonian with long-range (power-law) interactions is investigated by means of the replica theory. Both ferromagnetic and antiferromagnetic interactions are considered. The use of continuous variables instead of Ising variables in the spherical version of the model allows us to calculate the free energy exactly. The existence of an equilibrium phase transition is investigated based on the replica-symmetric solution.

The results are applied to the Coulomb-glass model of interacting localized electrons in a disordered solid. This model is shown not to have an equilibrium phase transition for spatial dimensions $D \leqslant 4$. For $D > 4$ the model has a phase transition to an ordered phase; however, it does not have a phase transition to a 'glassy' phase.

1. Introduction

Classical Ising Hamiltonians with random fields are widely used in physics to model disordered systems. In disordered magnetic systems the Ising variable describes a spin and in disordered electronic systems with localized states (Pollak 1970, Efros and Shklovskii 1975, for a recent review see Pollak 1992) the Ising variable describes the occupation of a lattice site with an electron. The theoretical investigation of thermodynamic properties for random-field systems is a difficult problem, even in the case of nearest-neighbour interactions (Imry and Ma 1975, Imbrie 1984, Fisher 1986, Bricmont and Kupiainen 1987, 1988). The long-range character of the Coulomb interaction in the electronic system makes the problem much more difficult and has prevented a systematic study of the Coulomb system with a random field up to now. The question of whether the Coulomb system (which is sometimes called 'Coulomb glass') has an equilibrium phase transition to a 'glassy' phase, was first asked by Davies, Lee and Rice (1982, 1984) and Grünewald, Pohlmann, Schweitzer and Würz (1982, 1983). Their results based on numerical calculations of a modified Edwards–Anderson order parameter (Edwards and Anderson 1975) has been inconclusive with regard to a glass transition. Up to now the question whether there is an equilibrium phase transition or not has not been answered (Pollak 1992).

Therefore it would be very useful to have a more simple model, which can be solved exactly and retains some of the main characteristics of the original model. Such a model is the spherical version of the random-field model. Hornreich and Schuster

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0305-4470/93/122883+11$07.50

(1982) investigated this model in the case of short-range interactions by means of the replica theory to eliminate the random fields. Instead of Ising variables the spherical model has soft (continuous) variables as was first introduced by Berlin and Kac (1952) for the nearest-neighbour Ising model. Later the spherical model was applied to systems with long-range interactions (Joyce 1966) and to spin glasses (Kosterlitz, Thouless and Jones 1976). Recently there has been a renewed interest in the spherical model for disordered systems (Jagannathan and Rudnick 1989, Jagannathan, Eva and Rudnick 1991, Crisanti and Sommers 1992). The advantage of the spherical model is that the evaluation of the partition function is much easier because the trace over the variables becomes a multiple integral instead of a multiple sum in the case of Ising variables.

In this paper the spherical random field Hamiltonian with long-range interactions, both ferro- and anti-ferromagnetic, is investigated by means of the replica theory, the results are applied to the Coulomb-glass problem. The paper is organized as follows: the model Hamiltonian is introduced in section 2. Section 3 summarizes the calculation of the free energy of the spherical random field model first given by Hornreich and Schuster (1982). The corresponding saddle-point equations are treated on the replica-symmetric level. The existence of a phase transition in the system with ferromagnetic interactions is discussed in section 4, while section 5 deals with the anti-ferromagnetic models. Special attention is laid on the Coulomb-glass system of interacting localized electrons in section 6. The existence of a phase transition is proved depending on the spatial dimension of the system and the nature of the low-temperature phase is studied. Finally section 7 is devoted to some conclusions and the discussion of the results.

2. The model

The spherical random field model consists of $N = L^D$ variables S_i on the sites of a regular D-dimensional hypercubic lattice, which may represent, for instance, spins or electric charges. The model has pair interactions U_{ij} (which we take translational invariant for convenience) between the variables S_i and a quenched random field φ_i coupling linearly to the variables. The Hamiltonian of the model is given by:

$$H = \sum_i \varphi_i S_i + \tfrac{1}{2} \sum_{i \neq j} U_{ij} S_i S_j + \sum_i h_i S_i. \tag{1}$$

The field h is a symmetry-breaking field which is taken constant ($h_i = h$) for ferromagnetic interactions or alternating ($h_i = \pm h$ for the two sublattices) for anti-ferromagnetic interactions. The values φ_i of the random field are independent random variables with a Gaussian probability distribution:

$$W(\varphi) = \frac{1}{\sqrt{2\pi}\,\varphi_0} \exp\left[-\frac{\varphi^2}{2\varphi_0^2}\right]. \tag{2}$$

The dynamic variables S_i are continuous real variables ranging from $-\infty$ to ∞. To make the model well defined and to avoid states with diverging energy, a constraint on the values of the variables (the spherical constraint) is added:

$$\sum_i S_i^2 = \frac{N}{4}. \tag{3}$$

The trace of an operator $\hat{O}$ over the variables S_i is given by a multiple integral (instead of a multiple sum in the Ising case)

$$\mathrm{Tr}\,\hat{O} = \int_{-\infty}^{\infty} \mathrm{d}S_1 \ldots \int_{-\infty}^{\infty} \mathrm{d}S_N\, \delta\left(\frac{N}{4} - \sum_i S_i^2\right) \hat{O}. \tag{4}$$

The symmetric interaction matrix U has to be such that the energy is extensive (proportional to N) in the thermodynamic limit $N \to \infty$. It must especially have a well defined lowest eigenvalue independent of N in the thermodynamic limit $N \to \infty$ (which is necessary for the existence of a ground state with an extensive energy).

3. Calculation of the free energy of the system

In this section we briefly summarize the derivation of the free energy of the spherical random field model given by Hornreich and Schuster (1982).

For any fixed realization $\{\varphi_i\}$ of the random field the free energy is given by

$$\beta F(\{\varphi_i\}) = -\ln Z(\{\varphi_i\}) = -\ln \mathrm{Tr} \exp[-\beta H(\{\varphi_i\})]. \tag{5}$$

The basic quantity under consideration is the quenched average $\bar{F}$ of the free energy:

$$\beta \bar{F} = \beta \overline{F(\{\varphi_i\})} = -\int \prod_i (\mathrm{d}\varphi_i\, W(\varphi_i)) \ln Z(\{\varphi_i\}). \tag{6}$$

The calculation of $\bar{F}$ involves an average over the logarithm of the partition function. The usual way to overcome this difficulty is the so-called replica trick first used by Edwards (1970) in the study of polymer physics. One writes

$$\ln Z = \lim_{n \to 0} \frac{1}{n} (Z^n - 1) \tag{7}$$

and

$$Z^n = \prod_{\alpha=1}^{n} Z_\alpha = \mathrm{Tr}_\alpha \exp\left[-\beta \sum_{\alpha=1}^{n} H^\alpha(S_i^\alpha)\right]. \tag{8}$$

The H^α $(\alpha = 1, \ldots, n)$ may be interpreted as identical non-interacting replicas of the system. Tr_α is the trace over all variables S_i^α. Finally the free energy is calculated by interchanging the order of the φ-integrals, the trace and the limit $n \to 0$.

Using the Fourier-representation for the δ-functions originating from the spherical constraints in the trace (see equation (4)), the nth power of the partition function Z may be written as

$$Z^n(\{\varphi_i\}) = \left(\frac{1}{2\pi \mathrm{i}}\right)^n \prod_\alpha \left(\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty} \mathrm{d}z_\alpha\right) \prod_{i,\alpha} \left(\int_{-\infty}^{\infty} \mathrm{d}S_i^\alpha\right)$$
$$\times \exp\left[-\beta \sum_{i,\alpha} (\varphi_i + h_i) S_i^\alpha - \frac{\beta}{2} \sum_{i \neq j, \alpha} U_{ij} S_i^\alpha S_j^\alpha + \sum_\alpha z_\alpha \left(\frac{N}{4} - \sum_i S_i^{\alpha 2}\right)\right]. \tag{9}$$

Averaging equation (9) with respect to the random fields and carrying out the Gaussian integrals over the variables S_i^α yields

$$\overline{Z^n} = \left(\frac{1}{2\pi \mathrm{i}}\right)^n \prod_\alpha \left(\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty} \mathrm{d}z_\alpha\right) \exp[N \Psi(\{z_\alpha\})]. \tag{10}$$

with

$$\Psi(\{z_\alpha\}) = \sum_\alpha \frac{z_\alpha}{4} + \frac{\beta^2}{4N} \boldsymbol{h}\boldsymbol{V}^{-1}\boldsymbol{h} - \frac{1}{2N} \ln \det \boldsymbol{V}$$
$$V_{i\alpha j\alpha'} = z_\alpha \delta_{ij}\delta_{\alpha\alpha'} + \tfrac{1}{2}\beta U_{ij}\delta_{\alpha\alpha'} - \frac{\beta^2\varphi_0^2}{2}\delta_{ij} \tag{11}$$

where $\boldsymbol{h}$ is the field vector of the symmetry-breaking field. The factor N in the exponent of equation (10) allows the application of the saddle-point method ($\ln \det \boldsymbol{V}$ and $\boldsymbol{h}\boldsymbol{V}^{-1}\boldsymbol{h}$ are proportional to N for $N \to \infty$). In the thermodynamic limit $N \to \infty$ the free energy per site is given by

$$-\beta \bar{f} = -\frac{1}{N}\beta \bar{F} = \frac{1}{N} \lim_{n\to 0} \frac{1}{n} (\overline{Z^n} - 1) = \lim_{n\to 0} \frac{1}{n} \Psi(\{z_{\alpha 0}\}) \tag{12}$$

where $z_{\alpha 0}$ is the saddle point of the exponent in equation (10). The saddle point has to be determined from

$$\frac{\partial}{\partial z_\alpha} \Psi(\{z_\beta\})|_{z_\beta = z_{\beta 0}} = 0 = \frac{1}{4} + \frac{1}{4N}\beta^2 \boldsymbol{h} \frac{\partial}{\partial z_\alpha} \boldsymbol{V}^{-1}\boldsymbol{h} - \frac{1}{2N}\frac{\partial}{\partial z_\alpha} \operatorname{Tr} \ln \boldsymbol{V}. \tag{13}$$

We now diagonalize the matrix $\boldsymbol{V}$ with respect to the site index i by applying the Fourier transformation, which involves diagonalizing the interaction matrix $\boldsymbol{U}$. The eigenvalues of $\boldsymbol{U}$ are denoted by u_k. The matrix $\boldsymbol{V}$ is now given by

$$V_{k\alpha k'\beta} = z_\alpha \delta_{kk'}\delta_{\alpha\beta} + \tfrac{1}{2}\beta u_k \delta_{kk'}\delta_{\alpha\beta} - \frac{\beta^2\varphi_0^2}{2}\delta_{kk'}. \tag{14}$$

To proceed further we assume that all $z_{\alpha 0}$ are equal to z at the saddle point; that means we choose the replica-symmetric solution. In this case $\boldsymbol{V}$ can be diagonalized in the replica-index simply by applying the Fourier transformation to $\boldsymbol{V}$. If we insert these results into equation (13) and take the limit $n \to 0$ equation (13) simplifies to the final form of the saddle-point equation in this limit (with $z = \beta s$):

$$0 = \tfrac{1}{4}\beta - \frac{\frac{1}{4}\beta h^2}{(s + \frac{1}{2}u_{k0})^2} - \frac{1}{2N}\sum_k \left[\frac{1}{s + \frac{1}{2}u_k} + \frac{\frac{1}{2}\beta\varphi_0^2}{(s + \frac{1}{2}u_k)^2} \right]. \tag{15}$$

Here u_{k0} is the eigenvalue of the interaction matrix belonging to $\boldsymbol{k} = 0$ for ferromagnetic interactions or $\boldsymbol{k} = (\pi, \ldots, \pi)$ for anti-ferromagnetic interactions. Whereas the first term in the $\boldsymbol{k}$-sum also arises in the corresponding equation of the system without random field, the second term is the result of the random field. Equation (15) reduces to the correct result for the system without random field in the limit $\varphi_0 \to 0$.

We now want to study the existence of a phase transition in the system without field ($h = 0$) in general. Therefore we assume that the $\boldsymbol{k}$-sum in equation (15) may be transformed into an integral over the eigenvalues of the matrix $\boldsymbol{U}$. With $P(u)$ being the distribution of the eigenvalues of u of the matrix $\boldsymbol{U}$, the saddle-point equation reads

$$\tfrac{1}{4}\beta = \tfrac{1}{2} \int \mathrm{d}u\, P(u) \left[\frac{1}{s + \frac{1}{2}u} + \frac{\frac{1}{2}\beta\varphi_0^2}{(s + \frac{1}{2}u)^2} \right]. \tag{16}$$

In order to look for a phase transition we have to check, whether equation (16) has a solution for all temperature or not. The real part of s has to be larger than $-\frac{1}{2}u_{\min}$—this is the condition that ensures that the Gaussian integral over S_i^α in equation (9) may

be carried out in the limit $n \to 0$. If the integral on the right-hand side diverges for $s \to -\frac{1}{2}u_{\min}$ the saddle-point equation (16) has a solution for all temperatures. Then the free energy and its derivatives are continuous functions of the temperature and no phase transition occurs. If the integral converges, however, a normal saddle point only exists for temperatures larger than a certain critical temperature T_c. For temperatures below T_c the saddle point sticks at its critical value. This sticking of the saddle point corresponds to a phase transition of the system (for a detailed discussion of this mechanism see for instance Berlin and Kac (1952)). In general one can distinguish three cases:

1. *The integral of the first term in equation (16), which corresponds to the pure* system, diverges for $s+\frac{1}{2}u_{\min} \to 0$. Then the integral of the second term diverges even stronger. Equation (16) has a solution for all temperatures and neither the pure system nor the system with random field has an equilibrium phase transition.

2. The integral of the first term converges whereas that of the second term, which contains a stronger singularity, diverges. In this case the pure system has a phase transition; however, any small disorder (any non-zero φ_0) destroys the transition.

3. Both integrals converge. Then the pure system has a transition; however, it depends on the magnitude φ_0 of the random field whether the saddle-point equation of the system with random field has a solution for all temperatures or not. If the *magnitude* φ_0 *fulfils the equation*

$$\lim_{s \to \frac{1}{2}u_{\min}} \tfrac{1}{2} \int \mathrm{d}u\, P(u) \frac{\frac{1}{2}\varphi_0^2}{(s+\frac{1}{2}u)^2} > \frac{1}{4} \tag{17}$$

a normal saddle point exists for all temperatures and no phase transition occurs. If the integral converges towards a value smaller than $\frac{1}{4}$ the saddle point sticks for low temperatures and the system with random field also has a phase transition.

In the case of a finite field $h \neq 0$ *the saddle-point equation has a normal solution* for any temperature and no phase transition occurs.

4. Ferromagnetic interactions

In the following we investigate a spherical random field system with long-range ferromagnetic interactions

$$U_{ij} = \frac{U_0}{r_{ij}^{\sigma}} \qquad U_0 < 0,\ \sigma > D. \tag{18}$$

The interaction exponent σ has to be larger than D to ensure that the ground state energy is proportional to N. The symmetry-breaking field is given by $h_i = h$. Then the saddle-point equation reads

$$0 = \tfrac{1}{4}\beta - \frac{\frac{1}{4}\beta h^2}{(s+\frac{1}{2}U_0 g(D, k=0))^2} - \frac{1}{2}\left(\frac{1}{2\pi}\right)^D \times \int_0^{2\pi} \mathrm{d}k_1 \ldots \int_0^{2\pi} \mathrm{d}k_D \left[\frac{1}{s+\frac{1}{2}U_0 g(D,k)} + \frac{\frac{1}{2}\beta\varphi_0^2}{(s+\frac{1}{2}U_0 g(D,k))^2}\right] \tag{19}$$

where $g(D, k)$ is the D-dimensional lattice sum

$$g(D,k) = \sum_{r \neq 0} \frac{1}{|r|} \exp(\mathrm{i}kr) \qquad r = (x_1, \ldots, x_D),\ k = (k_1, \ldots, k_D). \tag{20}$$

Table 1. Values of the interaction exponent σ, for which a phase transition occurs (ferromagnetic interactions).

Dimension D	Pure system	Random field system
1	$1<\sigma<2$	$1<\sigma<\frac{3}{2}$
2	$2<\sigma<4$	$2<\sigma<3$
3	$3<\sigma$	$3<\sigma<\frac{9}{2}$
4	$4<\sigma$	$4<\sigma<6$
5	$5<\sigma$	$5<\sigma$

This lattice sum may be calculated by the method of Nijboer and de Wette (1957). It may be written as

$$g(D,k,\sigma)=\frac{1}{\Gamma(\frac{1}{2}\sigma)}\left[\sum_{l_1,\ldots,l_D=-\infty}^{\infty}{}' \frac{\Gamma(\frac{1}{2}\sigma,\pi l^2)}{|l|^\sigma}\cos k_1 l_1\ldots\cos k_D l_D-\frac{2\pi^{\sigma/2}}{\sigma}\right.$$
$$\left.+\pi^{\sigma-D/2}\sum_{l_1,\ldots,l_D=-\infty}^{\infty}|l-k/2\pi|^{\sigma-D}\Gamma\left(-\frac{(\sigma-D)}{2},\pi(l-k/2\pi)^2\right)\right]. \qquad (21)$$

In order to look for a phase transition we have to check the convergence of the integrals in the saddle-point equation (19) for $s\to s_{\min}=-\frac{1}{2}U_0 g_{\max}$. To do this we expand the lattice sum around its maximum, that is around $k=0$:

$$g(D,k,\sigma)-g(D,0,\sigma)\sim|k|^{\sigma-D}+O(k^2) \qquad (\text{for } D<\sigma<D+2). \qquad (22)$$

For higher exponents σ, $g(D,k,\sigma)$ has a regular maximum at $k=0$, that means $g(D,k,\sigma)-g(D,0,\sigma)\sim k^2$ (for $\sigma=D+2$ a logarithmic correction arises). Hornreich and Schuster (1982) assumed a regular maximum in the spectrum of the interaction matrix in their discussion of the spherical random field model. While this is correct for short-range interactions, the assumption fails for long-range (power law) ferromagnetic interactions.

Given the behaviour of the lattice sum for small k it is easy to study the existence of a phase transition. Inserting equation (22) into the integrals in the saddle-point equation yields the following behaviour. The system without random field has a phase transition for $D<\sigma<2D$ for dimensions $D\leqslant 2$. It has a transition even for short-range interactions (and therefore for any $\sigma>D$) for dimensions $D>2$. These results have already been derived by Joyce (1966). The system with a finite random field undergoes a phase transition for $D<\sigma<3D/2$ for dimensions $D\leqslant 4$. It has a transition even for short-range interactions (and therefore for any $\sigma>D$) for dimensions $D>4$. In the random field system the transition only occurs if the strength of the random field is small enough. The maximum value of φ_0 is determined by equation (17). The values of the interaction exponent σ for which a phase transition occurs in the pure system (without random field) and may occur in the random field system are summarized in table 1.

5. Anti-ferromagnetic interactions

In this section we investigate a spherical random field system with long-range antiferromagnetic interactions

$$U_{ij}=\frac{U_0}{r_{ij}^\sigma} \qquad U_0>0,\ \sigma>0. \qquad (23)$$

The symmetry-breaking field $\boldsymbol{h}$ is a staggered field $h_i = \pm h$, corresponding to the two sublattices. The saddle-point equation and the lattice sum $g(D, \boldsymbol{k}, \sigma)$ are again given by equations (19) and (20). However, in the anti-ferromagnetic case $U_0 > 0$ the minimal value of s is given by $s_{\min} = -U_0 g(D, \boldsymbol{k}_0, \sigma)$ with $\boldsymbol{k}_0 = (\pi, \ldots, \pi)$. To study the convergence of the integrals in the saddle-point equation for $s \to s_{\min}$ we have to expand the lattice sum around the minimum point $\boldsymbol{k} = (\pi, \ldots, \pi)$. This point is a regular minimum in any dimension and, for any interaction exponent $\sigma > 0$, that means

$$g(D, \boldsymbol{k}, \sigma) - g(D, 0, \sigma) \sim |\boldsymbol{k}|^2 + O(|\boldsymbol{k}|^3) \qquad \text{(for any } D\text{)}. \tag{24}$$

It follows that the spherical model with long-range anti-ferromagnetic interactions shows the same behaviour as the model with short-range interactions (Hornreich and Schuster 1982). The pure system (without random field) has no phase transition for $D \leqslant 2$, it has a transition for $D > 2$. The system with random field has no phase transition for $D \leqslant 4$, it has a transition for $D > 4$, provided that the strength of the random field is small enough (see equation (17)). These results are independent of the value of the interaction exponent σ.

6. The Coulomb glass

The random-field model with long-range anti-ferromagnetic interactions is especially interesting, because for $\sigma = 1$ it is identical to the so-called Coulomb glass. The Coulomb-glass model was first proposed by Efros and Shklovskii (1975) to describe the influence of the Coulomb interaction on the properties of localized electronic states in disordered insulators, for instance, in amorphous or doped crystalline semiconductors. The long-range correlations caused by the Coulomb interaction are expected to strongly influence both equilibrium and non-equilibrium (such as transport) properties of the system. It was noticed by Davies *et al* (1982, 1984) and by Grünewald *et al* (1982, 1983) that there is a clear analogy between this model and a random field Ising model or a spin glass. They proposed that the electronic system may undergo a glass transition (which clarifies the origin of the term 'Coulomb glass'). In these investigations order parameters were defined, which are similar to that of Edwards and Anderson (1975) for spin glasses. The new definitions, however, try to overcome the difficulties arising in the original definition in the presence of a random field. These order parameters were calculated numerically either by a Monte Carlo method (Davies *et al* 1982, 1984) or by solving local mean-field equations (Grünewald *et al* 1982, 1983). Although the values of the order parameters were found to increase from zero to a finite value with decreasing temperature, the question, whether there is a sharp transition or not, could not be answered definitely. The results were therefore inconclusive with respect to an equilibrium phase transition.

In this section we discuss the behaviour of the Coulomb glass model using the results of section 5. The Coulomb-glass model consists of $N = L^D$ sites of a regular hypercubic lattice. Each site may be occupied by a (localized) electron or not. The electrons couple to the random field φ_i and interact via an unscreened Coulomb potential, hopping terms are completely neglected due to strong localization. To preserve the charge neutrality each site of the lattice has to possess a compensating background charge. Here the particle-hole symmetric case is considered, that means the compensating charge has to be $-e/2$. In this case the chemical potential is zero

in the grand canonical ensemble. The Hamiltonian of the model is given by

$$H=\tfrac{1}{2}\sum_{i\neq j}\frac{U_0}{r_{ij}}n_in_j+\sum_i(\varphi_i+h_i)n_i \qquad n_i=\pm\tfrac{1}{2} \tag{25}$$

We want to study the spherical version of the Coulomb-glass model; that means we treat the occupation numbers n_i as continuous variables and add the spherical constraint

$$\sum_i n_i^2=\frac{N}{4}. \tag{26}$$

The resulting model is exactly a spherical random field model with long-range antiferromagnetic interactions and $\sigma=1$. It follows that the spherical version of the Coulomb glass has no equilibrium phase transition for $D\leqslant 4$. It undergoes a phase transition for $D>4$, provided that the strength of the random field is small enough (see equation (17)).

In order to clarify the nature of the transition we calculate the mean staggered occupation number ν, which is defined by

$$\nu=\frac{1}{L^D}\sum_{j_1,\ldots,j_D}(-1)^{j_1+\ldots+j_D}n_{j_1,\ldots,j_D} \tag{27}$$

where the variables $j_1,\ldots,j_D$ count the lattice sites in the D spatial directions. The staggered occupation number ν may be obtained from the free energy by

$$\nu=\lim_{h\to 0}\frac{\mathrm{d}}{\mathrm{d}h}f=\lim_{h\to 0}\left(\frac{\partial}{\partial h}f+\frac{\partial}{\partial s}f\bigg|_{s=s_0}\frac{\mathrm{d}s_0}{\mathrm{d}h}\right). \tag{28}$$

For finite h the saddle-point equation has always a normal solution and the second term within the brackets is always zero because of the saddle-point condition. The first term can be calculated easily:

$$\nu=-\frac{h}{2(s+\frac{1}{2}U_0g(D,k_0))}. \tag{29}$$

We will now discuss the solutions of the saddle-point equation in the limit $h\to 0$. Above the transition temperature of the model without field the saddle-point equation has a normal solution (where the denominator in equation (29) is finite) for $h=0$, therefore the staggered occupation number ν goes to zero proportional to h in the limit $h\to 0$. Below the transition temperature the saddle-point equation has no solution for $h=0$. In the limit $h\to 0$ we have

$$s+\tfrac{1}{2}U_0g(D,k_0)\sim h \qquad \text{for } h\to 0 \tag{30}$$

and the staggered occupation number ν has a finite value depending on β. A more detailed study (see the appendix of this paper) shows

$$\nu\sim\sqrt{1-\frac{T}{T_c}} \tag{31}$$

below the critical temperature. Obviously the above phase transition is a transition to an ordered phase with an alternating occupation of the sites. It is therefore no (electron) glass transition.

7. Discussion and outlook

In this paper the spherical random field model with long-range (power-law) interactions is studied. The free energy of the system can be calculated exactly using the replica trick to eliminate the random fields. The saddle-point equations of the spherical model are treated on the replica-symmetric level. As usual in the spherical-model calculations the existence of a phase transition depends on the convergence properties of an integral over the eigenvalues of the interaction matrix. The behaviour of the ferromagnetic model is essentially different from that with anti-ferromagnetic interactions. The ferromagnetic model undergoes a phase transition in any dimension D for certain values of the interaction exponent σ and small enough φ_0. For $D>4$ the ferromagnetic model has a phase transition for any σ. There is no simple dimensional shift that connects the behaviour of the random field model in D dimensions with that of the model without random field in $D-2$ dimensions. (Such a relation only exists if the spectrum of the interaction matrix has a regular minimum.) In contrast, the model with anti-ferromagnetic interactions has no phase transition for $D \leq 4$ for any interaction exponent σ. For $D>4$ it has a transition for any σ. The model with long-range anti-ferromagnetic interactions shows the same behaviour as a model with short-range interactions and the relation $D_l^{\text{pure}}+2=D_l^{\text{random}}$ that connects the lower critical dimensions of the systems with and without random field is valid.

These results are applied to the spherical version of the Coulomb-glass model which is a special case of the spherical random field model with anti-ferromagnetic interactions and $\sigma=1$. Consequently the model has no equilibrium phase transition for spatial dimensions $D \leq 4$ for any strength of the random field. For $D>4$ the model has a phase transition for a small enough random field. In this case the low-temperature phase is an ordered phase corresponding to an alternating occupation of the lattice sites. A 'glassy' phase does not exist for the spherical Coulomb-glass model in any dimension. This result may be seen as a hint that the original Coulomb-glass model (with Ising variables representing the occupation of the lattice sites) does not have a phase transition to a glassy state either. (It has to be pointed out here that we deal with an equilibrium phase transition; the question of whether the Coulomb glass has a dynamic glass transition (for a review see Pollak 1992) is not a subject of this paper.)

It is interesting to compare the results of this paper with the work on a spherical version of spin glass (having random interactions instead of random fields) by Kosterlitz, Thouless and Jones (1976), who found an equilibrium phase transition to a spin glass state. The question arises, whether the result of this paper is modified if the model has not only a random field but also structural disorder due to the random positions of impurities in a crystal which results in a random contribution to the interactions. A detailed study of this question as well as an investigation of possible replica-symmetry breaking remain tasks for the future.

Acknowledgments

The author wishes to thank the Sonderforschungsbereich 262 for financial support and the members of the Arbeitskreis Theoretische Chemie in the Institut für Physikalische Chemie der Universität Mainz for their hospitality during a six month visit, where parts of the work were done. The author would like to acknowledge helpful discussions with W John, A Möbius, M Schreiber and K Tenelsen.

Appendix. Calculation of the staggered occupation number ν

In this appendix we study the case that the system has an equilibrium phase transition and investigate the behaviour of ν in the low-temperature phase of the system with $h=0$. We first consider a finite magnitude h of the staggered field and then take the limit $h\to 0$. Below the transition temperature of the system with $h=0$, the saddle-point value of s is in lowest order in h given by

$$s_0=-\tfrac{1}{2}U_0 g(D,k_0)+t(\beta)h \tag{32}$$

where the influence of the temperature is contained only in the factor $t(\beta)$. Inserting equation (32) into the saddle-point equation for the system with finite field h yields

$$0=\tfrac{1}{4}\beta-\frac{1}{4}\frac{\beta h^2}{t^2(\beta)h^2}-A(h)-\tfrac{1}{2}\beta\varphi_0^2 B(h) \tag{33}$$

where $A(h)$ and $B(h)$ are given by

$$\begin{aligned} A(h)&=\frac{1}{2}\left(\frac{1}{2\pi}\right)^D\int_0^{2\pi}\mathrm{d}k_1\ldots\int_0^{2\pi}\mathrm{d}k_D\,\frac{1}{\frac{1}{2}U_0(g(D,k)-g(D,k_0))+t(\beta)h} \\ B(h)&=\frac{1}{2}\left(\frac{1}{2\pi}\right)^D\int_0^{2\pi}\mathrm{d}k_1\ldots\int_0^{2\pi}\mathrm{d}k_D\,\frac{1}{[\frac{1}{2}U_0(g(D,k)-g(D,k_0))+t(\beta)h]^2}. \end{aligned} \tag{34}$$

The limiting values of $A(h)$ and $B(h)$ for $h\to 0$ are denoted by A_0 and B_0. They are finite, because $A(h)$ and $B(h)$ are essentially the integrals on the right-hand side of the saddle-point equation. These integrals must converge for $s\to-\frac{1}{2}U_0 g(D,k_0)$ to get a phase transition. In the limit $h\to 0$ equation (33) reads

$$\frac{1}{t^2(\beta)}=1-2\varphi_0^2 B_0-\frac{4A_0}{\beta}. \tag{35}$$

Inserting equations (32) and (35) into equation (29) for the staggered occupation number ν gives

$$|\nu(\beta)|=\frac{1}{2t(\beta)}=\tfrac{1}{2}\sqrt{1-2\varphi_0^2 B_0-4A_0/\beta}=\nu_0\sqrt{1-\frac{T}{T_c}} \tag{36}$$

with

$$\nu_0=\tfrac{1}{2}\sqrt{1-2\varphi_0^2 B_0}\qquad kT_c=\frac{1-2\varphi_0^2 B_0}{4A_0}. \tag{37}$$

These results hold for any temperature below T_c, not only for temperatures near T_c. The critical exponent β has its usual spherical model value $\frac{1}{2}$. The critical strength of the random field above which the phase transition disappears may be derived from equation (36). It is given by $T_c(\varphi_c)=0$ from which follows $\varphi_c^2=\frac{1}{2}B_0$. This result may also be derived directly from the general equation (17). The zero-temperature value ν_0 of the staggered occupation number depends on φ_0. This result is due to the use of continuous variables in the spherical model.

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